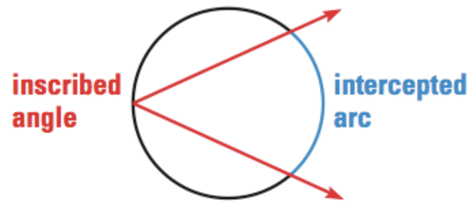
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

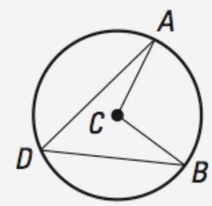


THEOREM

THEOREM 10.8 *Measure of an Inscribed Angle*

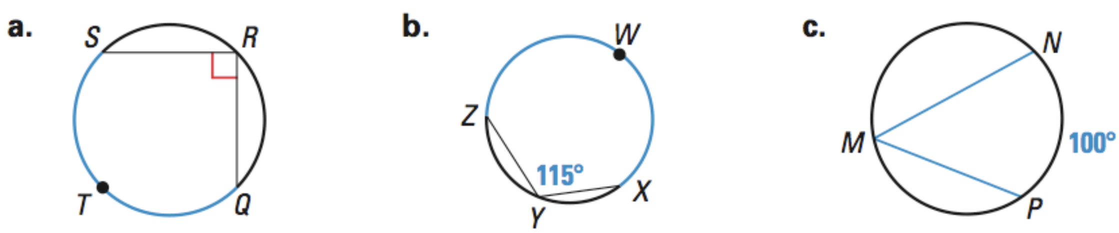
If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

$$m\angle ADB = \frac{1}{2}m\widehat{AB}$$



EXAMPLE 1 *Finding Measures of Arcs and Inscribed Angles*

Find the measure of the blue arc or angle.

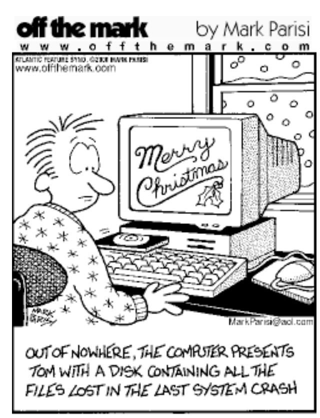


SOLUTION

a. $m\widehat{QTS} = 2m\angle QRS = 2(90°) = 180°$

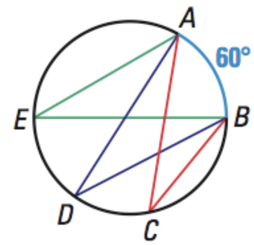
b. $m\widehat{ZWX} = 2m\angle ZYX = 2(115°) = 230°$

c. $m\angle NMP = \frac{1}{2}m\widehat{NP} = \frac{1}{2}(100°) = 50°$



EXAMPLE 2 Comparing Measures of Inscribed Angles

Find $m\angle ACB$, $m\angle ADB$, and $m\angle AEB$.



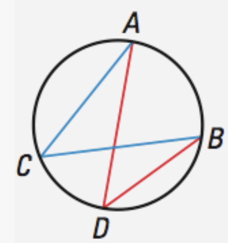
SOLUTION

The measure of each angle is half the measure of $\overset{\frown}{AB}$. $m\overset{\frown}{AB} = 60°$, so the measure of each angle is 30°.

THEOREM

THEOREM 10.9

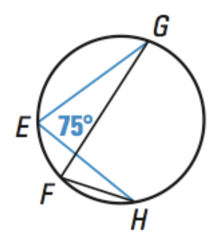
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle C \cong \angle D$

EXAMPLE 3 Finding the Measure of an Angle

It is given that $m\angle E = 75°$. What is $m\angle F$?



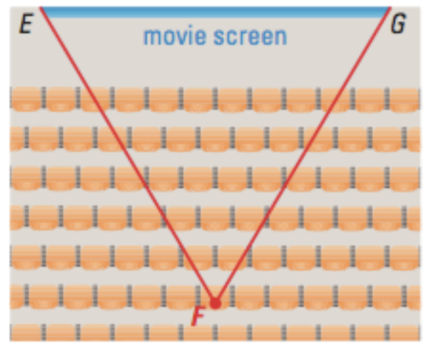
SOLUTION

$\angle E$ and $\angle F$ both intercept $\overset{\frown}{GH}$, so $\angle E \cong \angle F$.

▶ So, $m\angle F = m\angle E = 75°$.

EXAMPLE 4 Using the Measure of an Inscribed Angle

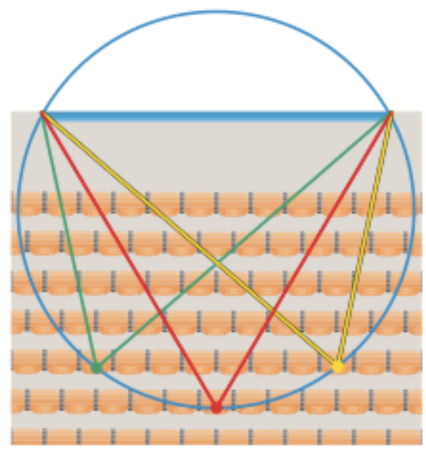
THEATER DESIGN When you go to the movies, you want to be close to the movie screen, but you don't want to have to move your eyes too much to see the edges of the picture. If E and G are the ends of the screen and you are at F, $m\angle EFG$ is called your *viewing angle*.



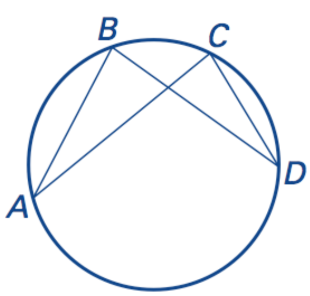
You decide that the middle of the sixth row has the best viewing angle. If someone is sitting there, where else can you sit to have the same viewing angle?

SOLUTION

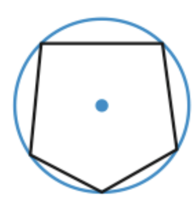
Draw the circle that is determined by the endpoints of the screen and the sixth row center seat. Any other location on the circle will have the same viewing angle.



6. Guided Practice: If $m\overset{\frown}{AD} = 164°$. What is $m\measuredangle C$?



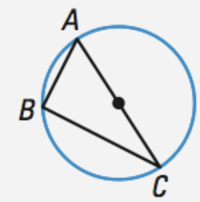
If all of the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an *inscribed polygon* and the circle is a *circumscribed circle*. You are asked to justify Theorem 10.10 and part of Theorem 10.11 in Exercises 39 and 40. A complete proof of Theorem 10.11 appears on page 840.



THEOREMS ABOUT INSCRIBED POLYGONS

THEOREM 10.10

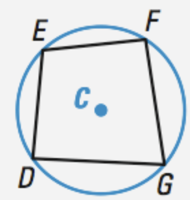
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$\angle B$ is a right angle if and only if $\overline{AC}$ is a diameter of the circle.

THEOREM 10.11

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

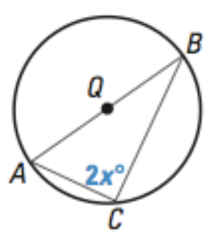


D, E, F, and G lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$.

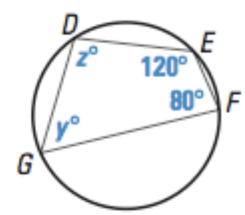
EXAMPLE 5 Using Theorems 10.10 and 10.11

Find the value of each variable.

a.



b.



SOLUTION

a. $\overline{AB}$ is a diameter. So, $\angle C$ is a right angle and $m\angle C = 90°$.

$$2x° = 90°$$

$$x = 45$$

b. $DEFG$ is inscribed in a circle, so opposite angles are supplementary.

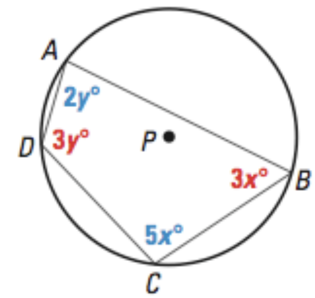
$$m\angle D + m\angle F = 180° \qquad m\angle E + m\angle G = 180°$$

$$z + 80 = 180 \qquad 120 + y = 180$$

$$z = 100 \qquad y = 60$$

EXAMPLE 6 ***Using an Inscribed Quadrilateral***

In the diagram, $ABCD$ is inscribed in $\odot P$. Find the measure of each angle.



SOLUTION

$ABCD$ is inscribed in a circle, so opposite angles are supplementary.

$3x + 3y = 180$ $\quad$ $5x + 2y = 180$

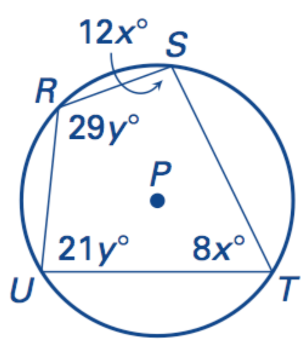
To solve this system of linear equations, you can solve the first equation for y to get $y = 60 - x$. Substitute this expression into the second equation.

$5x + 2y = 180$	Write second equation.
$5x + 2(60 - x) = 180$	Substitute $60 - x$ for y.
$5x + 120 - 2x = 180$	Distributive property
$3x = 60$	Subtract 120 from each side.
$x = 20$	Divide each side by 3.
$y = 60 - 20 = 40$	Substitute and solve for y.

▶ $x = 20$ and $y = 40$, so $m\angle A = 80°$, $m\angle B = 60°$, $m\angle C = 100°$, and $m\angle D = 120°$.

Guided Practice

7. In the diagram RSTU is inscribed in $\bigcirc P$. Find the measure of each angle.



11. What are inscribed angles?

12. Explain why the diagonals of a rectangle inscribed in a circle must be the diameters.